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# Jet Noise Predictions Based on Two Different Forms of Lilley's Equation

## Part 1: Basic Theory

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This report is a formal draft or working paper, intended to solicit comments and ideas from a technical peer group.

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## Abstract

The far field acoustic spectra at 90° to the downstream axis of some typical high speed jets are calculated from two different forms of Lilley's equation combined with some recent measurements of the relevant turbulent source function. These measurements, which were limited to a single point in a low Mach number flow, were extended to other conditions with the aid of a highly developed RANS calculation. The results are compared with experimental data over a range of Mach numbers. Both forms of the analogy lead to predictions that are in fair agreement with the experimental data at subsonic Mach numbers. The agreement is not quite as good at supersonic speeds, but the data appears to be slightly contaminated by shock- associated noise in this case.

## 1. Introduction

The acoustic analogy introduced by Lighthill (ref. 1) over 50 years ago remains the principle tool for predicting the noise from high speed air jets. In its most general formulation, it amounts to rearranging the Navier-Stokes equations into a form that separates out the linear terms and associates them with propagation effects that can then be determined as part of the solution. The non-linear terms are treated as "known" source functions to be determined by modeling and, in more recent approaches, parameterized with the parameters being determined from a steady RANS calculation. The "base" flow (about which the linearization is carried out) is usually assumed to be parallel and the resulting equation is usually referred to as Lilley's (ref. 2) equation.

The major drawback with these approaches is that the unsteady effects, which actually generate the sound, must be included as part of the model. This clearly puts severe demands on the modeling aspects of the prediction, which usually amount to assuming a functional form for the two point time delayed velocity correlation spectra. These predictions should, however, be less sensitive to the details of the model when it is possible to neglect variations in retarded time across the source correlation volume. It is therefore fortunate that this seems to be a reasonable approximation when performed in an appropriate moving frame of reference (ref. 3). The source models are usually tested by comparing them with measurements of the far field acoustic spectrum at 90° to the downstream jet axis, which is believed to be uninfluenced by propagation effects. The main purpose of this paper is to show that this spectrum can be accurately predicted by using an appropriate acoustic analogy approach combined with some measurements of the source function that were recently carried out by Harper-Bourne (ref. 4).

## 2. The Acoustic Analogy Equation and its Far-Field Solution

Reference 5 shows that the Navier-Stokes equations can be rewritten (for an ideal gas) as the linearized Navier-Stokes (LNS) equations about a very general "base flow" but with different (in general non-linear) dependent variables, with the heat flux vector replaced by a generalized enthalpy flux and with the viscous stresses replaced by a generalized Reynolds stress. *This is a true acoustic analogy (in the Lighthill (ref. 1) sense) in that it shows that there is an exact analogy between the flow fluctuations in any real flow and the linear fluctuations about a very general "base flow" due to an externally imposed "viscous" stress and "heat flux" vector.*

When the "base" flow is taken to be the unidirectional transversely sheared mean flow

$$v_i = \delta_{i1}\, U\left(x_2, x_3\right),\ \rho = \bar{\rho}\left(x_2, x_3\right),\ p = \bar{p} = \text{constant} \tag{1}$$

where $\boldsymbol{x} = \left\{x_1, x_2, x_3\right\}$ is a Cartesian coordinate system, $\boldsymbol{v} = \left\{v_1, v_2, v_3\right\}$ denotes the velocity, $p$ the pressure and $\rho$ the density, the resulting LNS equations can be combined to obtain the modified Lilley's (ref. 1) equation

$$Lp_e' = \left(\frac{D}{Dt}\frac{\partial}{\partial x_i}\widetilde{c^2} - \frac{\partial U}{\partial x_i} 2\widetilde{c^2}\frac{\partial}{\partial x_1}\right)\frac{\partial e_{ij}'}{\partial x_j} - \left(\gamma - 1\right)\frac{D^2}{Dt^2} Q \tag{2}$$

where

$$L \equiv \frac{D}{Dt}\left(\frac{\partial}{\partial x_i}\widetilde{c^2}\frac{\partial}{\partial x_i} - \frac{D^2}{Dt^2}\right) - 2\frac{\partial U}{\partial x_j}\frac{\partial}{\partial x_1}\widetilde{c^2}\frac{\partial}{\partial x_j} \tag{3}$$

is the variable-density Pridmore-Brown (ref. 17) operator

$$\widetilde{c^2} \equiv \gamma\, \bar{p}/\bar{\rho}\left(x_2, x_3\right) \tag{4}$$

is the square of the mean-flow sound speed, $\gamma$ = the specific heat ratio, $t$ denotes the time,

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1} \tag{5}$$

denotes the convective derivative based on $U$. The dependent variable $p_e'$ and source strengths $e_{ij}'$ and $Q$ are given by

$$p_e' \equiv p' + \frac{\gamma - 1}{2}\rho v'^2 \tag{6a}$$

and

$$e_{ij}' \equiv -\rho v_i' v_j' + \frac{\gamma - 1}{2}\delta_{ij}\rho v'^2 + \sigma_{ij} \tag{7a}$$

$$Q \equiv \frac{\partial}{\partial x_j}\eta'_j + \frac{\partial U}{\partial x_j}e'_{1i} \tag{8a}$$

where

$$\eta'_i \equiv -\rho v'_i h'_0 - q_i + \sigma_{ij} v'_j \tag{9}$$

when the limiting form of the general equations are used directly. Here

$$v'_i \equiv v_i - \delta_{i1} U \tag{10}$$

$$h'_o \equiv h' + \frac{1}{2} v'^2 \tag{11}$$

and $h' \equiv h - \dfrac{\widetilde{c^2}}{\gamma - 1}$, $h'_0 \equiv h' + \dfrac{v'^2}{2}$, with $h$ being the enthalpy. The viscous stress and heat flux vector, $\sigma_{ij}$ and $q_i$ respectively, are believed to play a negligible role in the sound generation process and will therefore be neglected in the following.

But the limiting form of the equations can also be rearranged to obtain a simpler result (ref. 19), which amounts to replacing the dependent variable $p'_e$ and source strengths $e'_{ij}$ and $Q$ by

$$p'_e \equiv p' \tag{6b}$$

$$e'_{ij} \equiv -\rho v'_i v'_j \tag{7b}$$

and

$$Q \equiv -\frac{\partial}{\partial x_j}\rho v'_j h' + v'_j \frac{\partial p'}{\partial x_j} \tag{8b}$$

in the inviscid limit. Th*ese results are exact consequences of the Navier-Stokes (inviscid Navier-Stokes) equations, but it is* frequently argued that the fluctuating enthalpy flux $\eta_i'$ ,which appears in the first formulation (and correspond to the isentropic part of the pressure density source in the Lighthill approach (ref. 1)) is only important for hot jets (refs. 2, 6 to 8) except, perhaps, at small angles to the downstream jet axis (ref. 9) Similar arguments would suggest that the enthalpy fluctuation term that appears in the second formulation is also unimportant in cold jets and since it can be shown that the ideal gas law implies

$$p' = \frac{\gamma - 1}{\gamma}\rho h' + \frac{\rho'}{\gamma}\widetilde{c^2} \tag{12}$$

it would seem that the second member in (8b) would also be negligible as well. We therefore neglect these terms in the present study. The main difference between these two formulations is then the dipole-like term $\dfrac{\partial U}{\partial x_i}(\gamma - 1)\dfrac{D^2}{Dt^2}e'_{1i}$ that appears in the first formulation, but not in the second. Since both formulations are initially exact consequences of the Navier-Stokes (inviscid Navier-Stokes) equations, any differences in the resulting acoustic predictions must be attributable to the introduction of these approximations and this research was initially undertaken as an attempt to distinguish between the two.

In either case, the generalized Lilley's equation can be solved (ref. 10) in terms of the free space Greens' function (ref. 18) $G\left(\boldsymbol{x},t\middle|\,\boldsymbol{y},\tau\right)$, which satisfies

$$LG\left(\boldsymbol{x},t\middle|\,\boldsymbol{y},\tau\right)=\delta\left(\boldsymbol{x}-\boldsymbol{y}\right)\delta\left(t-\tau\right) \tag{13}$$

and has outgoing wave behavior at infinity, to obtain the following expression

$$\overline{p^2\left(\boldsymbol{x},t\right)}=\int_{-\infty}^{\infty}\iint_V \overline{\gamma}_{ijkl}\left(\boldsymbol{x}\middle|\boldsymbol{y};\xi+\hat{\mathbf{i}}U_c\tau,t+\tau\right)R_{ijkl}^{M}\left(\boldsymbol{y};\xi,\tau\right)d\boldsymbol{y}\,d\boldsymbol{\xi}\,d\tau \tag{14}$$

for the pressure autocovariance (ref. 22) (notice that *the* $p_e'$ *in the first formulation* reduces to $p'$ in the far field)

$$\overline{p^2}\left(\boldsymbol{x},t\right)\equiv\frac{1}{2T}\int_{-T}^{T}p_e'\left(\boldsymbol{x},t_0\right)p_e'\left(\boldsymbol{x},t_0+t\right)dt_0 \tag{15}$$

where $V$ denotes integration over all space, $T$ denotes some large but finite time interval.

$$\overline{\gamma}_{ijkl}\left(\mathrm{x}\middle|\mathrm{y};\eta,t+\tau\right)\equiv\int_{-\infty}^{\infty}\gamma_{ij}\left(\boldsymbol{x},\boldsymbol{y},t_1+t+\tau\right)\gamma_{kl}\left(\boldsymbol{x}\middle|\boldsymbol{y}+\boldsymbol{\eta},t_1\right)dt_1 \tag{16}$$

where the propagation factor $\gamma_{kl}\left(\boldsymbol{x}\middle|\boldsymbol{y},t\right)$ is defined in reference 10. $R_{ijkl}^{M}\left(\boldsymbol{y};\xi,\tau\right)$ is a moving frame correlation tensor, which is defined in terms of the fixed frame density-weighted, fourth-order, two-point, time-delayed fluctuating velocity correlation (with the indicated arguments referring to all three terms preceding the parentheses)

$$R_{ijkl}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)\equiv\frac{1}{2T}\int_{-T}^{T}\rho v_i'v_j'\left(\boldsymbol{y},\tau_0\right)\rho v_k'v_l'\left(\boldsymbol{y}+\boldsymbol{\eta},\tau_0+\tau\right)d\tau_0 \tag{17}$$

and the second order fixed frame density weighted correlation

$$R_{ij}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)\equiv\frac{1}{2T}\int_{-T}^{T}\sqrt{\rho}v_i'\left(\boldsymbol{y},\tau_0\right)\sqrt{\rho}v_j'\left(\boldsymbol{y}+\boldsymbol{\eta},\tau_0+\tau\right)d\tau_0 \tag{18}$$

by

$$R_{ijkl}^{M}\left(\boldsymbol{y};\xi,\tau_0\right)\equiv R_{ijkl}\left(\boldsymbol{y};\xi+\hat{\mathbf{i}}U_c\tau,\tau\right)-R_{ij}\left(\boldsymbol{y};\mathbf{0},0\right)R_{kl}\left(\boldsymbol{y}+\xi+\hat{\mathbf{i}}U_c\tau;\mathbf{0},0\right) \tag{19}$$

where

$$\xi \equiv \boldsymbol{\eta} - \hat{\mathbf{i}} U_c \tau \tag{20}$$

denotes a moving frame coordinate system, with $U_c$ being the convection velocity of the turbulence.

Our interest is in the far field spectrum

$$I_\omega(\boldsymbol{x}) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t}\, \overline{p^2}(\boldsymbol{x}, t)\, dt \tag{21}$$

which can be calculated by taking the Fourier transform of eq. (14) and using the convolution theorem (ref. 18) to obtain

$$I_\omega(\boldsymbol{x}|\boldsymbol{y}) = 2\pi \int_{-\infty}^{\infty} \int_V \Gamma_{ij}(\boldsymbol{x}|\boldsymbol{y};\omega)\Gamma_{kl}^*\left(\boldsymbol{x}\middle|\boldsymbol{y} + \xi + \hat{\boldsymbol{i}} U_c \tau;\omega\right) e^{-i\omega\tau} R_{ijkl}^M(\boldsymbol{y}, \xi, \tau)\, d\xi\, d\tau \tag{22}$$

where

$$\Gamma_{ij} \equiv \frac{1}{2\pi} \int_{-\infty} e^{i\omega(t-\tau)} \gamma_{ij}(\boldsymbol{x}|\boldsymbol{y}, t-\tau)\, d(t-\tau) \tag{23}$$

is the Fourier transform of $\gamma_{ij}$ (we use capital letters to denote Fourier transform of the corresponding lower case quantity) and we introduced $I_\omega(\boldsymbol{x}|\boldsymbol{y})$,the acoustic spectrum at $\boldsymbol{x}$ due to a unit volume of turbulence at $\boldsymbol{y}$, i.e.,

$$I_\omega(\boldsymbol{x}) = \int_V I_\omega(\boldsymbol{x}|\boldsymbol{y})\, d\boldsymbol{y} \tag{24}$$

in order to simplify the formulas. The relevant far field expansion of $\Gamma_{ij}$ is given in reference 10.

The only approximations made up to this point is the neglect of the enthalpy and viscous source terms, but equation (22) will depend on the turbulent source correlations only through

$$\mathcal{R}_{ijkl}(\boldsymbol{y}, \tau) \equiv \int_V R_{ijkl}^M(\boldsymbol{y}, \xi, \tau)\, d\xi \tag{25}$$

if variations in retarded time across the correlation volume are neglected, i.e., if $\Gamma_{kl}^*\left(\boldsymbol{x}\middle|\boldsymbol{y} + \boldsymbol{\xi} + \hat{\boldsymbol{i}} U_c \tau;\omega\right)$ is assumed to be constant over the correlation volume (ref. 3). However, the definitions (15) and (19) imply that the integration variable in equation (25) can be changed back to $\boldsymbol{\eta}$, which means that

$$\mathcal{R}_{ijkl}(\boldsymbol{y}, \tau) \equiv \int_V \left[R_{ijkl}(\boldsymbol{y}, \boldsymbol{\eta}, \tau) - R_{ij}(\boldsymbol{y}; \mathbf{0}, 0)\, R_{kl}(\boldsymbol{y} + \boldsymbol{\eta}; \mathbf{0}, 0)\right] d\boldsymbol{\eta} \tag{26}$$

Equation (22) can now be written more simply as

$$I_\omega\left(\boldsymbol{x}|\boldsymbol{y}\right)\to\left(\frac{2\pi}{x}\right)^2\frac{2\pi\omega}{c_\infty}\sin\theta\,\bar{\Gamma}_{ij}\left(\boldsymbol{x}|\boldsymbol{y}_\perp\right)\bar{\Gamma}^*_{kl}\left(\boldsymbol{x}|\boldsymbol{y}_\perp\right)\Phi^*_{ijkl}\left(\boldsymbol{y};\omega\left(1-M_c\cos\theta\right)\right) \quad \text{as } x\to\infty \tag{27}$$

where

$$\Phi_{ijkl}\left(\boldsymbol{y},\omega\right)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\tau}\mathcal{R}_{ijkl}\left(\boldsymbol{y},\tau\right)d\tau \tag{28}$$

is the spectral tensor of the source correlation and

$$M_c\equiv U_c/c_\infty \tag{29}$$

is the convective Mach number of the turbulence. This result shows that it is only necessary to model the overall spectral tensor itself and not the detailed two-point time delayed correlations of the turbulence. However, the radiated sound should still be relatively insensitive to the detailed turbulence structure even when the latter quantities are modeled (as is at least partially done below). This would not be the case if the moving frame had not been introduced before neglecting the retarded time variations (ref. 3).

For reasons given in the introduction, our interest here is in the spectrum at 90° to the jet axis where $\cos\theta = 0$.We only carry out the analysis for the first formulation and simply give the final result for the second ( for reasons that will become clear when that is actually done ). Reference 10 shows that

$$\begin{aligned}I_\omega\left(\boldsymbol{x}|\boldsymbol{y}\right)=\frac{\left(\omega/c_\infty\right)^4}{\left(4\pi x\right)^2}&\left[\frac{x_ix_j}{x^2}-\frac{\gamma-1}{2}\delta_{ij}+\frac{i\left(\gamma-1\right)\delta_{1i}}{\omega}\frac{\partial U}{\partial y_j}\right]\left[\frac{x_kx_l}{x^2}-\frac{\gamma-1}{2}\delta_{kl}\right.\\&\left.-\frac{i\left(\gamma-1\right)}{\omega}\delta_{1k}\frac{\partial U}{\partial y_l}\right]\Phi^*_{ijkl}\left(\boldsymbol{y};\omega\right)\qquad\text{for }\theta=\pi/2.\end{aligned} \tag{30}$$

when $\widetilde{c_0^2}=c_\infty^2=$constant , i.e., in the isothermal case.

## 3. The Quasi-Normal and Axisymmetric Turbulence Approximations

To proceed further, we need to know something about the source spectral tensor $\Phi_{ijkl}$. The usual approach (refs. 12, 13, and 3) is to begin by assuming that the turbulence is quasi-normal (ref. 16) (see ref. 10) in order to obtain some relations among its components. It then follows that (see comments preceding equation (20))

$$\begin{aligned}R_{ijkl}\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right)-R_{ij}\left(\boldsymbol{y};\mathbf{0},0\right)R_{kl}\left(\boldsymbol{y}+\boldsymbol{\eta};\mathbf{0},0\right)=R_{ik}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)R_{jl}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)\\+R_{il}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)R_{jk}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)\end{aligned} \tag{31}$$

To further reduce the number of independent components it is usual to assume some kinematically possible symmetric form for the second order correlations. Early studies (ref. 20) assumed the turbulence to be isotropic, but that turns out to be incompatible with the Harper-Bourne (ref. 17) measurements (to be introduced below). The simplest assumption compatible with his results is the one introduced in references 12 and 13, namely that the turbulence is axisymmetric, which implies that (ref. 16)

$$R_{ij}\left(\boldsymbol{y};\boldsymbol{\eta},\tau\right)=A_0\eta_i\eta_j+B_0\delta_{ij}+C_0\delta_{1i}\delta_{1j}+D_0\left(\delta_{1j}\eta_j+\delta_{1j}\eta_i\right) \tag{32}$$

where $A_0$, $B_0$, $C_0$, and $D_0$ are functions of $\boldsymbol{y}$, $\tau_0$, and $\eta_\perp$; $A_0$, $B_0$, and $C_0$ are even functions $\eta_\perp$ and $D_0$ is an odd function of this quantity. This model is chosen because it is the most general of those whose mathematical properties have been studied in the literature and because it reflects the fact that the cross flow velocity components tend to be much more similar to one another than to the stream-wise component—even for non-axysymmetric flows. Inserting equation (32) into equation (31) and inserting the result into equation (30) via equations (28) and (26) yields (after a straight forward but tedious calculation that follows along the lines of the one in appendix A of reference 12)

$$I_\omega\left(\boldsymbol{x}|\mathbf{y}\right)\left(4\pi x\right)^2=2\left(\frac{\omega}{c_\infty}\right)^4\left[\Phi_1-(\gamma-1)\Phi_2+\left(\frac{\gamma-1}{2}\right)^2\Phi_3\right]+\left[(\gamma-1)\frac{\omega}{c_\infty}\left|\nabla M\right|\right]^2\Phi_4 \tag{33}$$

where

$$\Phi_1\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}\int_V R_{22}^2\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right)d\boldsymbol{\eta}\,d\tau \tag{34a}$$

$$\Phi_2\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}\int_V\left(R_{23}^2+R_{12}^2+R_{22}^2\right)d\boldsymbol{\eta}\,d\tau \tag{34b}$$

$$\Phi_3\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}\int_V\left(4R_{12}^2+2R_{23}^2+R_{11}^2+2\,R_{22}^2\right)d\boldsymbol{\eta}\,d\tau \tag{34c}$$

and

$$\Phi_4\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}\int_V\left({R_{12}}^2+R_{11}R_{22}\right)d\boldsymbol{\eta}\,d\tau \tag{34d}$$

are seemingly independent spectral functions. However, when compressibility effects are neglected (i.e., when $\rho$ is treated as a constant) the coefficients $A_0$, $B_0$, $C_0$, and $D_0$ are not all independent, but can be expressed in terms of two independent scalar functions of $\boldsymbol{y}$, $\tau_0$, $\eta_\perp$, and $\eta_1$, say $a$ and b (refs. 14 to 16), which scale like

$$b=\widetilde{\rho u_1^2}B\left(\tilde{\eta}_\perp,\tilde{\eta}_1\right)L_\perp^2/2 \tag{35}$$

and

$$g\equiv a-b_{\eta_1\eta_1}=\widetilde{\rho u_2^2}D\left(\tilde{\eta}_\perp,\tilde{\eta}_1\right) \tag{36}$$

where

$$\tilde{\eta}_1 \equiv \eta_1 / L_1 \qquad (37)$$

$$\tilde{\eta}_\perp \equiv \eta_\perp / L_\perp \qquad (38)$$

$L_1$ and $L_\perp$ denote characteristic stream-wise and transverse length scales of the turbulence, and $B$ and $D$ are $O(1)$ functions of the indicated arguments.

Turbulence measurements suggest

$$\varepsilon \equiv \frac{L_\perp}{4L_1} \qquad (39)$$

ought to be small. In fact, Harper-Bourne's (ref. 17) measurements (to be discussed below) suggest that $\varepsilon \simeq 2.7 \times 10^{-2}$.

Reference 10 shows that

$$\frac{\left(4/3\right)\Phi_1}{2\pi L_1 L_\perp^2 \left(\widetilde{\rho u_1^2}\right)^2} = \frac{\Phi_2}{2\pi L_1 L_\perp^2 \left(\widetilde{\rho u_1^2}\right)^2} = r^2 \int_{-\infty}^{\infty} e^{-i\omega\tau} \int_{-\infty}^{\infty}\int_{0}^{\infty} \left( \tilde{\eta}_\perp \frac{\partial D}{\partial \tilde{\eta}_\perp} \right)^2 \tilde{\eta}_\perp d\tilde{\eta}_\perp d\tilde{\eta}_1 d\tau \qquad (40a,b)$$

$$\frac{\Phi_3}{2\pi L_1 L_\perp^2 \left(\widetilde{\rho u_1^2}\right)^2} = \int_{-\infty}^{\infty} e^{-i\omega\tau} \int_{-\infty}^{\infty}\int_{0}^{\infty} \left[ \frac{\bar{B}^2}{8} + 2r^2 \left( \tilde{\eta}_\perp \frac{\partial D}{\partial \tilde{\eta}_\perp} \right)^2 \right] \tilde{\eta}_\perp d\tilde{\eta}_\perp d\tilde{\eta}_1 d\tau \qquad (40c)$$

$$\frac{\Phi_4}{2\pi L_1 L_\perp^2 \left(\widetilde{\rho u_1^2}\right)^2} = \frac{r}{2} \int_{-\infty}^{\infty} e^{-i\omega\tau} \int_{-\infty}^{\infty}\int_{0}^{\infty} \bar{B} \left( \frac{\partial}{\partial \tilde{\eta}_\perp} \tilde{\eta}_\perp{}^2 D \right) d\tilde{\eta}_\perp d\tilde{\eta}_1 d\tau \qquad (40d)$$

when $O(\varepsilon^2)$ terms are neglected-the ratio $r$ is defined by

$$r \equiv \widetilde{\rho u_2^2} \Big/ \widetilde{\rho u_1^2} \qquad (41)$$

Lacking any specific data to the contrary, it seems reasonable to suppose that

$$(\Gamma \bar{r})^2 = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-i\omega\tau} \int_V R_{22}^2 \left( \boldsymbol{y}, \boldsymbol{\eta}, \tau \right) d\boldsymbol{\eta} d\tau / \Phi_0 \qquad (42)$$

where

$$\Phi_0 \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-i\omega\tau} \int_V R_{11}^2 \left( \boldsymbol{y}, \boldsymbol{\eta}, \tau \right) d\boldsymbol{\eta}\, d\tau = \int_V H_0 \left( \boldsymbol{y}, \boldsymbol{\eta}, \tau \right) d\boldsymbol{\eta} \qquad (43)$$

is a constant, i.e., independent of ω and source location $y$. Equation (31) then becomes

$$I_\omega\left(\boldsymbol{x}\middle|\boldsymbol{y}\right)\left(2\pi x c_\infty\right)^2 = C_0^2\Phi_o\left(\boldsymbol{y},\omega\right)\left(\frac{\omega}{c_\infty}\right)^2\left[\omega^2+\left(\kappa\left|\nabla U\right|\right)^2\right] \tag{44}$$

where

$$2C_0^2 \equiv \frac{2}{3}(\Gamma\overline{r})^2\left[\frac{3}{4}-\left(\gamma-1\right)+2\left(\frac{\gamma-1}{2}\right)^2\right]+\frac{1}{2}\left(\frac{\gamma-1}{2}\right)^2 \tag{45}$$

is a constant and

$$\kappa \equiv \left(\frac{\gamma-1}{2}\right)\frac{1}{C_0}\sqrt{\frac{2\overline{r}\int\limits_{-\infty}^{\infty} e^{-i\omega\tau_0}\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}\overline{B}\left(\frac{\partial}{\partial\tilde{\eta}_\perp}\tilde{\eta}_\perp{}^2 D\right)d\tilde{\eta}_\perp d\tilde{\eta}_1 d\tau}{\int\limits_{-\infty}^{\infty} e^{-i\omega\tau_0}\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}\overline{B}^2\tilde{\eta}_\perp d\tilde{\eta}_\perp d\tilde{\eta}_1 d\tau}} \tag{46}$$

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The far field acoustic spectra at 90° to the downstream axis of some typical high speed jets are calculated from two different forms of Lilley's equation combined with some recent measurements of the relevant turbulent source function. These measurements, which were limited to a single point in a low Mach number flow, were extended to other conditions with the aid of a highly developed RANS calculation. The results are compared with experimental data over a range of Mach numbers. Both forms of the analogy lead to predictions that are in fair agreement with the experimental data at subsonic Mach numbers. The agreement is not quite as good at supersonic speeds, but the data appears to be slightly contaminated by shock-associated noise in this case.

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